

CBCS SCHEME

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18MAT21

Second Semester B.E. Degree Examination, Dec.2019/Jan.2020 Advanced Calculus and Numerical Methods

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ along $2\bar{i} - 3\bar{j} + 6\bar{k}$. (06 Marks)
- b. If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$ find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$. (07 Marks)
- c. Find the constants a and b such that $\vec{F} = (axy + z^3)\bar{i} + (3x^3 - z)\bar{j} + (bxz^2 - y)\bar{k}$ is irrotational. Also find a scalar potential ϕ if $\vec{F} = \nabla\phi$. (07 Marks)

OR

- 2 a. If $\vec{F} = xy\bar{i} + yz\bar{j} + zx\bar{k}$ evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve represented by $x = t, y = t^2, z = t^3, -1 \leq t \leq 1$. (06 Marks)
- b. Using Stoke's theorem Evaluate $\int_C \vec{F} \cdot d\vec{r}$ if $\vec{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ taken round the rectangle bounded by $x = 0, x = a, y = 0, y = b$. (07 Marks)
- c. Using divergence theorem, evaluate $\iiint_S \vec{F} \cdot \vec{n} \, ds$ if $\vec{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken around $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$. (07 Marks)

Module-2

- 3 a. Solve $(4D^4 - 8D^3 - 7D^2 + 11D + 6)y = 0$ (06 Marks)
- b. Solve $(D^2 + 4D + 3)y = e^{-x}$ (07 Marks)
- c. Using the method of variation of parameter solve $y'' + 4y = \tan 2x$. (07 Marks)

OR

- 4 a. Solve $(D^3 - 1)y = 3 \cos 2x$ (06 Marks)
- b. Solve $x^2y'' - 5xy' + 8y = 2 \log x$ (07 Marks)
- c. The differential equation of a simple pendulum is $\frac{d^2x}{dt^2} + W_0^2x = F_0 \sin t$, where W_0 and F_0 are constants. Also initially $x = 0, \frac{dx}{dt} = 0$ solve it. (07 Marks)

Module-3

- 5 a. Find the PDE by eliminating the function from $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$. (06 Marks)
- b. Solve $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ given $\frac{\partial z}{\partial y} = -2 \sin y$, when $x = 0$ and $z = 0$, when y is odd multiple of $\frac{\pi}{2}$. (07 Marks)
- c. Derive one-dimensional wave equation in usual notations. (07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg. 42+8 = 50, will be treated as malpractice.

OR

- 6 a. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$ given that when $x = 0$ $\frac{\partial z}{\partial x} = a \sin y$ and $z = 0$. (06 Marks)
- b. Solve $x(y - z) p + y(z - x) q = z(x - y)$. (07 Marks)
- c. Find all possible solution of $U_t = C^2 U_{xx}$ one dimensional heat equation by variable separable method. (07 Marks)

Module-4

- 7 a. Test for convergence for
 $1 + \frac{2!}{2^2} + \frac{3!}{3^2} + \frac{4!}{4^2} + \dots$ (06 Marks)
- b. Find the series solution of Legendre differential equation
 $(1 - x^2)y'' - 2xy' + n(n + 1) = 0$ leading to $P_n(x)$. (07 Marks)
- c. Prove the orthogonality property of Bessel's function as
 $\int_0^1 x \bar{j}_n(\alpha x) \bar{j}_n(\beta x) dx = 0 \quad \alpha \neq \beta$ (07 Marks)

OR

- 8 a. Test for convergence for
 $\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ (06 Marks)
- b. Find the series solution of Bessel differential equation $x^2 y'' + xy' + (n^2 - x^2)y = 0$ Leading to $\bar{j}_n(x)$. (07 Marks)
- c. Express the polynomial $x^3 + 2x^2 - 4x + 5$ interms of Legendre polynomials. (07 Marks)

Module-5

- 9 a. Using Newton's forward difference formula find $f(38)$. (06 Marks)

x	40	50	60	70	80	90
f(x)	184	204	226	250	276	304

- b. Find the real root of the equation $x \log_{10} x = 1.2$ by Regula falsi method between 2 and 3 (Three iterations). (07 Marks)
- c. Evaluate $\int_4^{5.2} \log x dx$ by Weddle's rule considering six intervals. (07 Marks)

OR

- 10 a. Find $f(9)$ from the data by Newton's divided difference formula:

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

- (06 Marks)
- b. Using Newton - Raphson method, find the real root of the equation $x \sin x + \cos x = 0$ near $x = \pi$. (07 Marks)
- c. By using Simpson's $\left(\frac{1}{3}\right)$ rule, evaluate $\int_0^6 \frac{dx}{1+x^2}$ by considering seven ordinates. (07 Marks)
